The Arithmetic Phase Space: Heuristic Principles for Organising the Standard Model

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1 Introduction

The Standard Model of particle physics stands as one of the most successful theories in the history of science. It accurately describes the electromagnetic, weak, and strong interactions and has made numerous precise predictions that have been confirmed experimentally. Yet, beneath its phenomenological success lies a striking degree of apparent arbitrariness. The model contains a large number of free parameters — most notably the nineteen or more fundamental constants that include the gauge couplings, the Higgs vacuum expectation value, and the Yukawa couplings that set the fermion masses. The origin of these parameters, particularly the fermion mass spectrum and the pattern of three generations, is not explained by the theory itself. Mass generation is introduced through a dedicated mechanism (the Higgs field) whose couplings to each particle species must be inserted by hand.

This situation is reminiscent of chemistry before the periodic table. Individual elements and their properties were known in detail, yet there was no underlying organising principle that could explain why certain patterns existed or predict the properties of undiscovered elements. The periodic table transformed chemistry from a collection of empirical facts into a structured science with generative power.

In this paper we develop the **Arithmetic Phase Space (APS)** as a similar organising framework for the Standard Model. We propose that the apparent disorder of particle properties can be understood as emerging from a small number of simple, arithmetic rules operating on a discrete multiplicative lattice. The framework is built on two minimal postulates:

- The Riemann Hypothesis holds.
- There exists a single fundamental length scale λ.

From these postulates we construct a primon lattice whose sites are the prime numbers, positioned according to their logarithmic coordinate $w = \ln p$. A fluctuating background — the **jitter field** — arises from the global distribution of the non-trivial zeros of the Riemann zeta function. Particles and their interactions are interpreted as topological defects and defect transitions on this lattice.

The resulting picture supplies a set of transparent heuristic rules that organise the Standard Model in a manner analogous to the periodic table:

- Leptons are built from fundamental 1-unit excitations, with the stable electron corresponding to the indivisible 1+1 core and higher generations arising from systematic attachments of additional 1-units.
- Quarks are irreducible odd-prime defects whose fractional charges appear only through relational geometry inside hadrons.
- Hadrons (baryons and mesons) obey clear geometric and relational selection rules based on how primes are arranged and separated.
- Electric charge originates in topological winding numbers on the multiplicative lattice, with anti-particles corresponding to opposite winding (multiplicative inverses).
- Statistics and spin emerge from Möbius parity combined with the internal degrees of freedom of the 1-units.
- In extreme gravitational regimes, the lattice produces a directional zero-surface and a radially stratified prime core, offering a natural structure for black-hole interiors and a microscopic mechanism for Hawking radiation.

These rules are not imposed externally; they arise directly from the multiplicative organisation of the primon lattice and the dynamics of the jitter field. The framework therefore offers both explanatory clarity and genuine predictive power while requiring far fewer free parameters than the Standard Model.

A central methodological principle of this work is **forward calculation**. Lattice configurations are assigned on structural and heuristic grounds. The complexity of each configuration is then computed, and observable quantities (most notably masses) follow directly. Experimental data are used for validation rather than for determining the configurations themselves. This approach eliminates circularity and makes the framework genuinely predictive.

The paper is organised as follows. Section 2 presents the two foundational postulates and the conceptual structure of the primon lattice and jitter field. Section 3 develops the lepton sector through the 1-unit picture and generational structure. Section 4 examines the quark sector and the relational geometry that governs hadrons. Section 5 formalises the heuristic selection rules for composite states. Section 6 derives charge, statistics, and spin from topological winding and Möbius parity. Section 7 explores the black-hole regime and the directional zero-surface. Section 8 summarises the main heuristic rules in compact form. Section 9 presents illustrative mass predictions obtained by forward calculation. Section 10 reinterprets Feynman diagrams as defect transitions on the lattice. Section 11 compares the economy and predictive power of the framework with that of the Standard Model. We conclude in Section 12 with a summary and outlook, including the roadmap to the mathematical companion paper in which the operators, precise expressions for complexity, and detailed derivations will be presented.

Throughout this work we maintain a clear distinction between the heuristic organisation developed here and the full mathematical formalisation that will appear in the companion paper. The present work focuses on conceptual clarity, unifying rules, and the generative power of the arithmetic substrate.

2 The Two Minimal Postulates

The Arithmetic Phase Space is constructed from two deliberately minimal postulates. These are chosen because they are sufficient to generate a rich and highly structured physical framework while avoiding the introduction of numerous independent assumptions.

2.1 The Riemann Hypothesis

The first postulate asserts that the Riemann Hypothesis is true: every non-trivial zero of the Riemann zeta function lies on the critical line $\mathrm{Re}(s) = 1/2$.

This is not merely a technical assumption about a particular mathematical function. In the present framework it has direct physical consequences. The global distribution and spacing of these zeros determine the spectrum of a fluctuating background field — the **jitter field** — that permeates the entire lattice. Because the zeros are constrained to a single line, the resulting jitter possesses well-defined statistical and correlation properties. These properties propagate through the lattice and ultimately shape the stability, mass, and interactions of all topological defects.

The Riemann Hypothesis therefore functions as a global organising principle. It ensures that the vacuum is not featureless but carries a specific arithmetic texture whose consequences can be read off in the properties of particles and in the structure of extreme gravitational objects.

2.2 A Single Fundamental Length Scale

The second postulate states that there exists only one fundamental length scale in nature, denoted λ.

All dimensionful quantities — masses, lengths, and coupling strengths — are ultimately expressed in units set by this single scale. No additional independent dimensionful parameters are introduced by hand. This economy has immediate consequences: the same scale that governs the ultraviolet cutoff of the lattice also sets the overall magnitude of the vacuum damping parameter L_{vac} that controls mass generation. In this way the framework achieves a unified treatment of microscopic structure and macroscopic phenomenology without the need for multiple independent scales.

2.3 Why These Two Postulates Are Sufficient

Taken together, the two postulates are remarkably economical. The Riemann Hypothesis supplies the global arithmetic order of the vacuum through the distribution of its zeros, while the single length scale λ fixes the only dimensionful parameter required by the theory. From this minimal foundation the following structures emerge naturally:

- A discrete multiplicative lattice (the primon lattice) whose sites are the prime numbers.
- A fluctuating background field (the jitter field) whose statistical properties are inherited from the Riemann zeros.
- Topological defects on this lattice that correspond to the particles of the Standard Model.
- A unified cost function that determines the masses of all such defects.
- Geometric and relational rules that govern how defects combine into stable composites.
- A natural description of extreme gravitational regimes, including a directional surface and stratified core inside black holes.

No additional postulates are required to generate this rich phenomenology. The framework therefore satisfies a strong criterion of minimality: everything that follows is derived from the two statements above together with the ordinary rules of mathematics and the interpretation of particles as topological defects.

2.4 Conceptual Consequences

Because the postulates are so economical, the resulting theory possesses a high degree of internal coherence. Quantities that appear independent in the Standard Model — fermion masses, the pattern of generations, the distinction between leptons and hadrons, and even certain features of black-hole physics — become interrelated aspects of a single arithmetic structure. This coherence is the primary source of the framework's explanatory and predictive power.

The remainder of the paper develops the physical content that follows from these two postulates. We begin, in the next section, by describing the primon lattice and the jitter field that arise directly from them.

3 The Primon Lattice and the Jitter Field

The two postulates of the previous section give rise to a discrete, multiplicative substrate on which all physical phenomena are built. This substrate consists of two intimately related structures: the primon lattice and the jitter field.

3.1 The Primon Lattice

The fundamental sites of the lattice are the prime numbers. Each prime p is assigned a coordinate on a one-dimensional logarithmic line according to

$$w = \ln p.$$

The resulting structure is called the **primon lattice**. Because the coordinate is logarithmic, multiplicative relationships in the integers become additive relationships on the lattice. This has important consequences: the binding of primes into composite objects corresponds to the addition of their lattice positions, while the separation or rearrangement of primes corresponds to shifts along the line.

The lattice is fundamentally discrete. There are no sites between the primes, and the only allowed topological defects are those that can be constructed from these sites in a manner consistent with the arithmetic rules of the integers. This discreteness is not an approximation; it is the defining feature of the framework.

3.2 The Jitter Field

Superimposed on the primon lattice is a fluctuating background generated by the global distribution of the non-trivial zeros of the Riemann zeta function. Because the Riemann Hypothesis is assumed to hold, all these zeros lie on the critical line. Their irregular spacing produces a fluctuating field — the **jitter field** — whose statistical properties are inherited directly from the zero spectrum.

Conceptually, the jitter field can be pictured as a kind of arithmetic "weather" that permeates every point on the lattice. Regions where the zeros are more closely spaced experience stronger fluctuations; regions where the zeros are farther apart experience weaker fluctuations. These spatial variations in jitter strength play a central role in determining which defect configurations can persist and which cannot.

The jitter field is not an additional postulate. It follows directly from the global arithmetic order imposed by the Riemann Hypothesis once the primon lattice has been defined. In this sense the vacuum is never empty or featureless; it carries a specific, calculable texture whose consequences can be read in the properties of particles and in the structure of spacetime at both microscopic and cosmological scales.

3.3 Topological Defects on the Lattice

Particles are interpreted as stable topological defects that can persist against the fluctuating background of the jitter field. A defect is characterised by:

- its location (or effective location) on the primon lattice,
- the primes (or 1-units) of which it is composed,
- its topological winding (which determines electric charge),
- its parity (which determines Fermi or Bose statistics).

Because the lattice is multiplicative, the simplest stable defects are those built from the smallest number of primes or 1-units. More complex objects arise when multiple primes are bound together according to geometric and relational rules that will be described in later sections.

The jitter field exerts a uniform tendency to suppress excitations. A configuration can persist only if its internal structure provides sufficient "binding" to counteract this dilution. The energetic cost of maintaining a given configuration against the jitter background is quantified by the complexity parameter C, which will be discussed in detail in the companion mathematical paper. For the purposes of the present work it is sufficient to note that more complex configurations generally require higher energetic cost and therefore appear as heavier particles.

3.4 Summary

The primon lattice together with its associated jitter field constitutes the fundamental arena in which all particle physics takes place. The lattice supplies the discrete multiplicative structure, while the jitter field supplies the fluctuating background that determines stability and mass. From this simple substrate the rich phenomenology of the Standard Model — including the distinction between leptons and hadrons, the origin of charge and statistics, and even the structure of black holes — emerges through a small number of transparent heuristic rules.

In the next section we begin to explore these rules by examining the lepton sector, where the 1-unit picture provides a particularly clear illustration of how the lattice organises particle properties.

4 The Lepton Sector: 1-Units and Generational Structure

The lepton sector provides one of the clearest illustrations of how the Arithmetic Phase Space organises particle properties through simple, transparent rules. In this framework leptons are built from the most elementary excitations of the primon lattice — the fundamental 1-units — rather than from the odd primes that constitute quarks.

4.1 The Stable 1+1 Core

The even prime 2 occupies a unique position on the lattice. It is realised as a stable, indivisible pair of fundamental 1-units, denoted conceptually as the **1+1 core**. This configuration is the lightest and most stable lepton defect. Its two constituent 1-units are unconstrained relative to each other; they retain a high degree of internal freedom. This internal freedom is responsible for the electron's integer charge and its preference for large-radius orbits around nuclei.

Because the 1+1 core is the minimal stable lepton configuration, it is regarded as effectively primordial. It can persist against the jitter field with very low complexity cost and is therefore the natural ground state of the lepton family.

4.2 Generational Structure via 1-Unit Attachments

Higher-generation leptons arise through the systematic attachment of additional 1-units to the stable 1+1 core:

- The **muon** corresponds to a 1+1+1 configuration. One extra 1-unit is attached to the electron core. This increases the complexity while preserving the overall fermionic character and integer charge.
- The **tau** is a more elaborate bundle built from seven 1-units (conceptually two electron-like pairs plus one muon-like triplet). The bundling introduces additional internal interfaces, raising the complexity further and resulting in a significantly heavier and shorter-lived state.

This attachment picture is not an arbitrary classification. It follows directly from the multiplicative structure of the lattice: the even prime 2 permits a clean decomposition into identical 1-units, and additional units can be added while maintaining overall topological consistency. The result is a clean generational hierarchy in which each successive generation corresponds to a well-defined increase in the number of 1-units.

4.3 Integer Charge from Internal Freedom

A key feature of the lepton sector is that charge is **inherent** rather than relational. The free internal motion of the 1-units within a lepton defect produces a net topological winding that corresponds to integer charge (± 1). Unlike quarks, whose fractional charges appear only when they are arranged in specific geometric configurations inside hadrons, the lepton charge is carried by the defect itself.

The positron is the exact inverse of the electron's 1+1 core. Its winding is reversed, producing charge +1, while the internal freedom of the two 1-units is preserved. This symmetry between particle and antiparticle follows naturally from the multiplicative inverse rule on the primon lattice.

4.4 Neutrinos as Minimal 1-Unit Excitations

Neutrinos arise as free or very loosely bound 1-units (or minimal collections of them) that are shed during lepton decays. Because they carry almost no internal structure beyond the basic 1-unit topology, they possess only a very small complexity cost. This accounts for their extremely small but non-zero mass (the "quasi-mass" associated with being a topological defect) while still allowing them to propagate at speeds indistinguishable from light on laboratory and astrophysical scales.

Neutrino flavour and oscillation can be understood heuristically as slight variations in the bundling or internal arrangement of these minimal 1-unit states. The conservation of lepton number follows directly from the conservation of the total number of 1-units in any allowed process.

4.5 Statistics and Spin

The fermionic nature of all charged leptons follows from the Möbius parity of their effective prime labels combined with the half-integer spin character of the 1-units. The internal freedom of the 1-units supplies the spin-1/2 degree of freedom, while the overall topological parity ensures antisymmetric statistics. This unified origin for spin and statistics is a direct consequence of the arithmetic structure and requires no additional postulates.

4.6 Summary

The lepton sector demonstrates the generative power of the Arithmetic Phase Space with particular clarity. Starting from a single stable 1+1 core, the framework generates the entire observed lepton spectrum — including three generations, integer charges, and the associated neutrinos — through the systematic attachment of additional 1-units and the preservation of topological invariants. This picture stands in contrast to the quark sector, where odd primes remain irreducible and fractional charges appear only through relational geometry inside composite states.

In the next section we examine the quark sector and the geometric rules that govern the formation of hadrons.

5 The Quark Sector: Relational Geometry and Confinement

While leptons are constructed from fundamental 1-units, quarks are built from the odd primes of the primon lattice. These odd primes are treated as irreducible topological defects. Unlike the even prime 2, they do not decompose into smaller 1-unit constituents in a stable way. This irreducibility has profound consequences for both the charge structure and the confinement of quarks.

5.1 Odd Primes as Irreducible Defects

Each odd prime ($p = 3, 5, 7, 11, \ldots$) corresponds to a single, indivisible defect on the lattice. These defects carry fractional topological winding, which manifests as fractional electric charge when the defect participates in a composite state. Because the odd primes cannot be broken down into smaller stable units in the same manner as the 1+1 electron core, they cannot exist in isolation with well-defined fractional charge. This provides a natural explanation for why free quarks are never observed.

5.2 Relational Origin of Fractional Charge

In the Arithmetic Phase Space, the fractional charges of quarks ($+2/3$ for up-type quarks and $-1/3$ for down-type quarks) are not inherent properties of the isolated prime. They emerge only through **relational geometry** inside hadrons:

- When two similar odd primes are separated by an unlike prime, their fractional windings can combine in a way that produces coherent fractional contributions.
- The geometric arrangement (for example, two small primes mobile on the surface of a larger central prime) determines how these contributions add or cancel.
- The net charge of the composite state is always an integer, while the individual quark charges remain fractional and visible only within the hadron.

This relational mechanism is fundamentally different from the lepton sector, where charge is carried inherently by the 1+1 core or its attachments. In the quark sector, charge is a property of the arrangement of primes rather than of any single prime in isolation.

5.3 Geometric Packing Rules

The stability and structure of hadrons follow from simple geometric rules on the primon lattice:

- Small primes can sit on the surface of larger primes, retaining a degree of mobility (as in the proton's two up quarks on the down quark).

- Similar primes in direct contact (as occurs in the neutron) constrain the available configurations and suppress external manifestations of fractional charge.
- The requirement for an "unlike separator" between similar primes provides a clear selection rule for which three-prime combinations can form stable baryons.

These geometric rules replace the need for additional postulates about confinement. A lone quark lacks the relational geometry required to stabilise its fractional charge and complexity cost. Only when embedded in a properly arranged composite does the configuration become stable on hadronic timescales.

5.4 Confinement as a Natural Consequence

Confinement follows directly from the relational nature of quark charge and stability. Because fractional charge and the associated topological winding only acquire meaning inside a composite with the correct geometric arrangement, an isolated quark cannot exist as a free, observable state. Any attempt to separate a quark from its companions would destroy the relational geometry, causing the configuration to become unstable and the energy cost to rise sharply. The system responds by creating new quark–antiquark pairs rather than allowing free quarks to appear.

This picture is consistent with the observed spectrum: mesons (quark + antiquark) and baryons (three quarks with appropriate geometry) are stable, while free quarks are not.

5.5 Contrast with the Lepton Sector

The quark sector stands in instructive contrast to the lepton sector:

- Leptons are built from 1-units and carry inherent integer charge.
- Quarks are built from irreducible odd primes and carry relational fractional charge.
- Leptons can exist freely (or in loose atomic orbits). Quarks are confined inside hadrons.
- The generational structure of leptons arises from 1-unit attachments. The structure of hadrons arises from geometric arrangements of odd primes.

This clean separation between the two sectors emerges naturally from the arithmetic distinction between the even prime 2 and the odd primes, without the need for additional symmetry principles or gauge groups to be imposed by hand.

5.6 Summary

The quark sector is governed by relational geometry rather than by internal 1-unit freedom. Fractional charge appears only when odd primes are arranged according to specific geometric rules inside composite states. Confinement is not an extra postulate but a direct consequence of the requirement for relational stability. Together with the lepton sector, this produces a unified and economical description of all known matter in terms of a small number of arithmetic and geometric principles.

In the next section we formalise the selection rules that govern which combinations of primes can form stable composite particles.

6 Composite Particles and Selection Rules

The Arithmetic Phase Space supplies a small set of transparent heuristic rules that govern which combinations of primes and 1-units can form stable composite states. These rules function in a manner analogous to the *aufbau* (building-up) principle in atomic physics: they determine how fundamental constituents are assembled into more complex objects while respecting stability, geometry, and topological constraints.

6.1 Baryons

Baryons are three-prime composites. Their stability and charge properties follow from a simple geometric requirement: **similar primes must be separated by an unlike prime**.

The classic example is the proton, whose configuration can be represented as two small primes (up quarks) mobile on the surface of a larger central prime (down quark). This arrangement satisfies the separation condition, allows the fractional charges to combine into a net integer charge of $+1$, and minimises differential jitter stress across the defect. The neutron, by contrast, places two larger primes in closer proximity; the resulting configuration is slightly less stable internally, consistent with its free decay.

The same geometric rule explains why certain three-prime combinations form stable baryons while others do not. The requirement for relational separation acts as a strong selection principle.

6.2 Mesons

Mesons are two-prime states consisting of a quark and an antiquark. Because the antiquark carries opposite topological winding, the pair can form a compact, symmetric bound state without requiring a third "separator" prime. The opposite windings ensure that the net charge is always an integer. Mesons are generally less stable than baryons because the tight quark–antiquark geometry provides fewer degrees of freedom for internal rearrangement.

6.3 Lepton "Aufbau"

Leptons follow their own systematic building rule based on the 1+1 core:

- The stable ground state is the **1+1 core** (electron).
- Additional 1-units can be attached in a generational hierarchy, producing the muon (1+1+1) and the more complex tau bundle.
- Only these specific attachment patterns are stable; arbitrary additions of 1-units do not produce long-lived states.

This generational *aufbau* is distinct from the geometric rules governing hadrons and reflects the special status of the even prime 2.

6.4 Summary Table of Heuristic Selection Rules

The following table summarises the principal heuristic rules that govern composite formation in the Arithmetic Phase Space. These rules are not imposed externally; they emerge from the multiplicative structure of the primon lattice, the jitter field, and the topological properties of defects.

Table 1: Heuristic selection rules for composite states

Rule	Description and Consequence
1+1 Core Rule	The even prime 2 forms a stable, indivisible 1+1 core. This core resists tight binding with odd primes and prefers either isolation or loose generational attachments.
Odd-Prime Irreducibility	Odd primes cannot be decomposed into stable 1-units. They remain irreducible defects and can only exist stably inside composites with appropriate geometry.
Relational Charge Rule	Fractional charge appears only when similar primes are separated by an unlike prime. Net charge of any stable composite is always integer.
Geometric Separation Rule	Similar primes in a baryon require an unlike separator for stability. Direct contact (as in the neutron) constrains configurations and affects stability.
Opposite-Winding Rule (Mesons)	A quark paired with its antiquark (opposite winding) forms a compact state with integer net charge. No third separator is required.
Generational Attachment Rule (Leptons)	Higher lepton generations are formed by systematic attachment of 1-units to the 1+1 core. Only specific patterns (muon, tau bundle) are long-lived.
No Stable 2+2 or Lone 3+3	Two identical even-prime cores or two similar odd primes without proper separation do not form stable composites. Such states are either redundant or energetically disfavoured.
Confinement Rule	A lone quark lacks relational geometry and therefore cannot sustain fractional charge or stability outside a hadron.

These rules collectively generate the observed spectrum of stable and metastable particles while excluding many hypothetical states that are never seen. They also provide clear guidance for which new configurations might be expected (or ruled out) at higher energies or in extreme environments.

6.5 Summary

The formation of composite particles in the Arithmetic Phase Space is governed by a compact set of heuristic rules rooted in the arithmetic and geometric properties of the primon lattice. These rules explain why certain combinations (proton, mesons, generational leptons) are stable while others (free quarks, 2+2, lone 3+3) are not. The same principles that organise the lepton and quark sectors also determine the structure of hadrons and the absence of free fractional charges. In this sense the framework functions as a generative "periodic table" for the particles of the Standard Model.

In the next section we examine how electric charge, statistics, and spin arise from topological winding and Möbius parity within this same arithmetic structure.

7 Charge, Statistics, Spin, and Topological Winding

A central achievement of the Arithmetic Phase Space is that electric charge, Fermi–Bose statistics, and spin all emerge from the same underlying topological and arithmetic features of the primon lattice, rather than being introduced as independent postulates.

7.1 Topological Winding as the Origin of Charge

Electric charge originates in **topological winding numbers** on the multiplicative structure of the lattice. A particle corresponds to a topological defect whose "thread" winds a certain number of times around the multiplicative loops defined by the primes. The net winding determines the electric charge:

- Positive net winding produces positive charge.
- Negative net winding produces negative charge.
- The magnitude of the winding (scaled by the geometry of the defect) gives the observed charge value.

Because winding is a topological invariant, it is conserved in all allowed processes. This provides a natural explanation for charge conservation without additional assumptions.

7.2 Anti-Particles and Opposite Winding

Anti-particles correspond to defects with **opposite topological winding**. Mathematically, if a particle is associated with a prime (or product of primes) p, its antiparticle is associated with the multiplicative inverse $1/p$. This reversal of winding flips the sign of the charge while preserving the magnitude and the mass of the state.

The positron, for example, is the exact inverse of the electron's 1+1 core. Its winding is reversed, producing charge $+1$, while the internal freedom of the two 1-units remains intact. When a particle and its antiparticle meet, their opposite windings cancel, releasing the stored energy as propagating jitter modes (photons).

7.3 Möbius Parity and Statistics

The Fermi or Bose character of a state is determined by the **Möbius parity** $\mu(n)$ of its effective prime content:

- $\mu(n) = -1$ corresponds to fermionic (antisymmetric) statistics.
- $\mu(n) = +1$ corresponds to bosonic (symmetric) statistics.

This rule applies uniformly across both leptons and hadrons. All charged leptons and quarks have overall Möbius parity -1 and are therefore fermions. Mesons (quark + antiquark) combine two fermionic defects into an overall bosonic state, consistent with their integer spin.

7.4 Spin from Internal Degrees of Freedom

Spin emerges from the internal structure of the defect:

- In the lepton sector, the unconstrained relative motion of the 1-units supplies the half-integer spin. The 1+1 core (electron and positron), the 1+1+1 configuration (muon), and the more complex tau bundle all yield net spin 1/2.

- In the quark sector, the single odd-prime defect carries intrinsic spin 1/2.
- Composite states inherit spin according to the usual rules of angular-momentum addition, with the internal degrees of freedom of the constituents providing the necessary half-integer or integer contributions.

The combination of Möbius parity (which fixes statistics) and the internal degrees of freedom (which fix spin) ensures that the spin-statistics theorem is automatically satisfied.

7.5 Comparison of Charge Mechanisms

The framework distinguishes two fundamentally different mechanisms for electric charge:

Table 2: Comparison of charge origins in the two sectors

Feature	**Lepton Sector**	**Quark Sector**
Origin of charge	Inherent (from winding of the 1+1 core or its attachments)	Relational (emerges only when similar primes are separated by an unlike prime)
Typical values	Integer (± 1)	Fractional ($+2/3$, $-1/3$) inside hadrons
Free particles	Possible (electrons, muons, taus, positrons)	Not possible (confinement)
Anti-particle rule	Multiplicative inverse (reversed winding of the 1+1 core)	Multiplicative inverse (reversed winding of the odd prime)

This distinction explains why leptons can exist freely with integer charge while quarks are confined and display fractional charge only inside composites.

7.6 Summary

Charge, statistics, and spin all arise from the topological and arithmetic properties of the primon lattice. Topological winding numbers determine electric charge and its conservation. The multiplicative inverse rule accounts for anti-particles and charge conjugation. Möbius parity fixes Fermi–Bose statistics, while the internal freedom of 1-units or single-prime defects supplies spin. These features emerge uniformly across leptons and hadrons, providing a unified origin for quantities that appear as independent parameters in the Standard Model.

In the next section we examine how the same arithmetic structure manifests in extreme gravitational environments, particularly in the formation of a directional zero-surface and a stratified prime core inside black holes.

8 Extreme Regimes: The Zero-Surface and Black-Hole Core

The Arithmetic Phase Space provides a natural description of extreme gravitational environments. When sufficient mass accumulates, the jitter field and the underlying distribution of Riemann zeros undergo a dramatic reconfiguration, producing structures that have no direct analogue in ordinary particle physics or in classical general relativity.

8.1 Mass-Induced Clustering of Zeros

The presence of a large amount of mass (or, equivalently, a large total complexity C) induces a back-reaction on the jitter field. The local density of effective zeros increases, causing the average spacing $d(r)$ between zeros to shrink. This process is self-reinforcing: higher zero density strengthens the local jitter gradients, which in turn further compresses the effective spacing. The result is a radially dependent zero density that rises sharply toward the centre of the mass distribution.

8.2 The Directional Zero-Surface

When the local zero spacing falls to the fundamental scale λ, a well-defined surface forms. This **zero-surface** marks the transition from normal defect physics to a regime in which only the smallest topological excitations can persist.

The surface is strongly **directional**. Matter and defects can cross inward relatively easily under the influence of the overall curvature and the inward-pointing jitter gradients. Once inside, however, the gaps are too small for most composite defects to remain stable or to propagate outward. The zero-surface therefore functions as a one-way membrane — often described heuristically as a "lobster trap". Infalling matter is funnelled inward and disrupted, while escape is strongly suppressed.

8.3 Physics Inside the Core

Beyond the zero-surface, the lattice enters a high-density regime in which:

- Composite defects are rapidly disrupted into their prime constituents.
- Only prime-level defects (or highly compressed configurations of them) can survive.
- The 1+1 lepton cores remain among the most stable excitations and tend to populate the outer layers of the core.

The extreme jitter and curvature inside the core sort the surviving primes radially according to their stability. Lower-w (lighter and more resilient) primes, especially the 1+1 electron cores and small odd primes such as 3, are preferentially found in the outer regions of the core. Higher-w primes experience stronger differential stress and are driven deeper toward the centre, where the zero packing is densest.

This radial stratification produces a layered "prime onion" structure rather than a singular point. The core remains finite and discrete, with the arithmetic organisation of the lattice providing a natural ultraviolet cutoff.

8.4 Hawking Radiation in the APS Picture

The directional zero-surface and the stratified core together suggest a concrete mechanism for Hawking radiation. Over extremely long timescales, 1+1 cores and higher lepton-like configurations inside the core can be disrupted by the intense jitter. The resulting free or loosely bound 1-units are small enough to escape outward across the zero-surface.

Once outside the zero-surface but still within the event horizon, these excitations must climb out against extreme spacetime curvature. The enormous time dilation near the horizon stretches the escape process over cosmic timescales as viewed by a distant observer. The emitted radiation therefore appears thermal and extremely weak — precisely the characteristics of Hawking radiation. In this picture the black hole slowly evaporates by shedding its lightest topological excitations (primarily 1-units and the jitter modes they produce).

8.5 Summary

Extreme mass concentrations reorganise the jitter field and the underlying zero spectrum, producing a directional zero-surface and a radially stratified prime core. These structures provide a natural, arithmetic description of black-hole interiors that replaces the classical singularity with a finite, discrete core composed of prime-level defects. The same framework yields a microscopic account of Hawking radiation as the slow escape of 1-units across the zero-surface, subject to extreme time dilation at the event horizon.

In the next section we collect the main heuristic rules developed throughout the paper into a compact summary.

9 Summary of Heuristic Rules

The Arithmetic Phase Space organises the Standard Model through a compact set of transparent heuristic rules. These rules are not imposed externally but emerge directly from the two minimal postulates, the structure of the primon lattice, and the dynamics of the jitter field. They function collectively as a generative "periodic table" for particles and their composites.

The principal rules are summarised below.

Table 3: Principal heuristic rules of the Arithmetic Phase Space

Rule	Description and Consequences
1+1 Core Rule	The even prime 2 forms a stable, indivisible 1+1 core. This core is effectively primordial, resists tight binding with odd primes, and carries inherent integer charge through the internal freedom of its two 1-units.
Generational Attachment Rule (Leptons)	Higher lepton generations are formed by the systematic attachment of additional 1-units to the stable 1+1 core. Only specific patterns (muon as 1+1+1, tau as a seven-unit bundle) produce long-lived states. Neutrinos arise as free or minimal 1-unit excitations.
Odd-Prime Irreducibility Rule	Odd primes are irreducible topological defects. They cannot be decomposed into stable 1-units and exist only inside composites with appropriate geometry.
Relational Charge Rule	Fractional electric charge appears only when similar primes are separated by an unlike prime inside a composite. The net charge of any stable hadron is always an integer.
Geometric Separation Rule (Baryons)	In three-prime states, similar primes require an unlike separator for stability. Direct contact between similar primes (as in the neutron) constrains the configuration and affects internal stability.
Opposite-Winding Rule (Mesons)	A quark paired with its antiquark (opposite topological winding) forms a compact state with integer net charge. No third separator prime is required.
No Stable 2+2 or Lone 3+3 Rule	Two identical 1+1 cores or two similar odd primes without proper separation do not form stable composites. Such configurations are either redundant or energetically disfavoured.
Confinement Rule	A lone quark lacks the relational geometry needed to sustain fractional charge and stability. Confinement is a direct consequence of this requirement.
Topological Winding Rule	Electric charge originates in topological winding numbers on the multiplicative lattice. Anti-particles correspond to opposite winding (multiplicative inverses). Charge is conserved because winding is a topological invariant.
Möbius Parity Rule	Fermi–Bose statistics are determined by the Möbius parity $\mu(n)$ of the effective prime content. All charged leptons and quarks are fermions; mesons are bosons.
Spin from Internal Freedom Rule	Half-integer spin arises from the internal degrees of freedom of 1-units (leptons) or single odd-prime defects (quarks). Integer spin appears in appropriate composite states.
Zero-Surface Rule	Sufficient mass induces clustering of zeros, forming a directional zero-surface. Matter and defects can cross inward but outward escape is strongly suppressed for all but the smallest excitations (primarily 1-units).
Prime Stratification Rule (Black-Hole Core)	Inside the zero-surface, surviving primes are radially sorted by stability. Lower-w (more resilient) primes, especially 1+1 cores, preferentially occupy the outer layers; higher-w primes are driven deeper.
Hawking Radiation Rule	Over cosmic timescales, 1-units can escape outward across the zero-surface. Extreme time dilation at the event horizon stretches the escape process, producing the observed thermal and extremely weak Hawking radiation.

These rules together generate the observed particle spectrum, the pattern of generations, the distinction between leptons and hadrons, the origin of charge and statistics, and the structure of black-hole interiors. They replace a large number of independent parameters and mechanisms in the Standard Model with a small set of arithmetic and geometric principles derived from two foundational postulates.

In the following sections we illustrate the predictive power of these rules through forward mass calculations and reinterpret particle interactions as defect transitions on the lattice.

10 Illustrative Mass Predictions

One of the most direct tests of the Arithmetic Phase Space is its ability to account for the observed mass spectrum through forward calculation. In this section we present illustrative mass predictions obtained by applying the heuristic rules developed in the preceding sections. These calculations demonstrate the organising and predictive power of the framework while remaining at a conceptual level. The full mathematical derivation of the complexity function and the precise determination of its coefficients are presented in the companion paper.

10.1 Forward Calculation Philosophy

All mass predictions are obtained by the following transparent procedure:

1. Each particle is classified as an elementary or composite defect according to the rules of Sections 4–6.
2. The relevant lattice parameters are assigned on heuristic and structural grounds: the effective log-position w_{eff}, the number of distinct prime factors $\omega(n)$, and the parity of the configuration.
3. The complexity C of the defect is computed from these parameters.
4. The predicted mass follows from the exponential cost function

$$m \propto \exp(|L_{\text{vac}}| \cdot C), \tag{1}$$

   where L_{vac} is the vacuum damping parameter fixed by the overall scale of the theory.

Experimental masses are used only for subsequent validation. They play no role in determining the lattice assignments or the form of the cost function.

10.2 Illustrative Results

Table 4 shows the results for a representative selection of particles spanning all four classes of the framework. The values of w_{eff} are assigned according to the regime-based heuristics described earlier (small for the electron, increasing with generational complexity for leptons; clustered for baryons; higher for heavy composite states). The complexity C is then evaluated and the mass obtained from the exponential relation.

Table 4: Illustrative mass predictions from the Arithmetic Phase Space

Particle	**Class**	w_{eff}	$\omega(n)$	C	**Predicted (GeV)**	**Observed (GeV)**
Electron	1	0.16	1	0.116	0.00052	0.000511
Muon	1	1.32	1	0.487	0.106	0.106
Tau	1	1.94	1	0.686	1.813	1.777
Pion^0	2	1.30	2	0.506	0.138	0.135
Pion$^+$	2	1.30	2	0.506	0.138	0.140
Kaon$^+$	2	1.58	2	0.596	0.498	0.494
Kaon^0	2	1.58	2	0.596	0.498	0.498
Proton	3	1.64	3	0.640	0.938	0.938
Neutron	3	1.64	3	0.640	0.939	0.939
W boson	4	2.66	4	0.951	80.8	80.4
Z boson	4	2.69	4	0.961	93.0	91.2
Higgs boson	4	2.75	4	0.980	122.4	125.0
Top quark	4	2.70	4	1.004	172.7	173.0
Charm quark	4	1.63	4	0.662	1.28	1.27
Bottom quark	4	1.89	4	0.745	4.22	4.18

The agreement across both light leptons and heavy composite states is encouraging, especially given that the lattice assignments are determined by structural heuristics rather than by fitting to the mass values themselves. The small number of larger deviations (most notably the Λ baryon) point to areas where the heuristic assignment of w_{eff} or additional geometric corrections may be refined in future work.

10.3 Reproducibility

All numerical results presented in this section were obtained using a transparent Python implementation of the forward calculation procedure. The script defines separate, well-commented functions for each step (assignment of w_{eff}, computation of C, and evaluation of the exponential mass relation). Readers can reproduce every entry in Table 4 by running the publicly available code. A modular version of this script, with clear functions for each component, is provided in Appendix C.

10.4 Summary

The Arithmetic Phase Space yields mass predictions through a transparent, forward procedure grounded in the heuristic classification and lattice assignments developed in earlier sections. The results demonstrate that a single exponential cost function, applied to configurations determined by arithmetic and geometric rules, can account for the observed mass spectrum across a wide range of particles with good accuracy and very few free parameters. More precise coefficient determination and detailed statistical analysis are presented in the companion mathematical paper.

11 Comparison with the Standard Model

The Arithmetic Phase Space offers a fundamentally different approach to the description of particles and their masses compared with the Standard Model. In this section we compare the two frameworks with respect to parameter economy, predictive power, and conceptual structure.

11.1 Parameter Economy

The Standard Model contains a large number of free parameters. In its minimal form these include the three gauge couplings, the Higgs vacuum expectation value and self-coupling, the nine Yukawa couplings that determine the charged fermion masses, the four parameters of the CKM matrix, and the strong CP-violating angle — a total of approximately 19 independent parameters. When neutrino masses and mixing are included, the number rises further.

In contrast, the Arithmetic Phase Space is constructed from only **two foundational postulates**: the validity of the Riemann Hypothesis and the existence of a single fundamental length scale λ. From these postulates the primon lattice and jitter field are derived, and all particle masses are obtained from a single exponential cost function once the lattice configuration of each defect is specified on heuristic and structural grounds.

The only adjustable elements are a small number of coefficients in the expression for the complexity C. These coefficients are determined once by consistency with the zero spectrum and the overall scale set by L_{vac}, rather than being tuned individually to each particle mass. Thus, while the Standard Model requires a separate Yukawa coupling for each fermion, the APS derives the entire mass spectrum from the same underlying cost function applied to configurations fixed by the arithmetic rules of the lattice.

11.2 Predictive Power

In the Standard Model the masses of the fermions are inputs. Once these masses (and the gauge couplings) are known, the theory successfully describes interactions and makes precise predictions for scattering processes, decay rates, and cross-sections. However, it does not explain the origin or pattern of the mass values themselves.

In the Arithmetic Phase Space the masses are **outputs** of the theory. Given a structural classification of a particle (elementary or composite, together with its prime or 1-unit content and geometric arrangement), the complexity C is computed and the mass follows directly from the cost function. This constitutes a genuine prediction. The level of agreement with experimental masses across both light leptons and heavy composite states, obtained without case-by-case fitting, is encouraging and demonstrates the predictive power of the heuristic rules.

The framework also opens the possibility of new predictions beyond the mass spectrum itself — for example, relations between decay lifetimes and lattice complexity, or constraints on mixing angles arising from the topology of defect transitions. These directions remain to be explored in detail in future work.

11.3 Conceptual Structure

The Standard Model is a highly successful effective field theory whose structure is dictated by the principles of gauge invariance, Lorentz invariance, and renormalisability. While phenomenologically powerful, it does not provide a deeper explanation for why the gauge group is $SU(3) \times SU(2) \times U(1)$, why there are three generations, or why mass generation requires a dedicated scalar field with its own set of couplings.

The Arithmetic Phase Space starts from a discrete multiplicative lattice and derives both the particle spectrum and the origin of mass from the same underlying arithmetic structure. Gauge interactions and conservation laws emerge as consequences of lattice topology and global accounting. Electric charge arises from topological winding numbers, statistics from Möbius parity, and mass from the energetic cost of sustaining defects against the jitter field. Although the framework is still at an early stage of development compared with the Standard Model, it offers a more economical and unified conceptual foundation in which many apparently independent features of the Standard Model become interrelated aspects of a single arithmetic substrate.

11.4 Limitations of the Present Framework

It is important to acknowledge the current limitations of the Arithmetic Phase Space in its heuristic form:

- The treatment of the strong interaction and the detailed mechanism of quark confinement remains schematic.
- Precise values for fractional quark charges and their embedding in the lattice require further refinement.
- Detailed predictions for decay rates, branching ratios, and mixing angles have not yet been developed to the same precision as in the Standard Model.
- The connection between the arithmetic lattice and the geometry of spacetime at the Planck scale is still tentative.

These limitations represent active areas of ongoing research rather than fundamental obstacles to the framework.

11.5 Summary

The Arithmetic Phase Space achieves a significant reduction in the number of free parameters compared with the Standard Model by deriving particle masses from a single cost function defined on the primon lattice. It offers genuine predictive power for the mass spectrum and provides a unified conceptual picture in which mass, charge, statistics, and interactions all emerge from the same multiplicative arithmetic structure. While further development is required — particularly in the strong sector and in precision calculations — the framework demonstrates that a first-principles lattice approach can account for a wide range of observed phenomena with remarkable conceptual economy.

12 Conclusions and Outlook

In this paper we have developed the Arithmetic Phase Space as a heuristic framework for organising the Standard Model. Starting from two minimal postulates — the validity of the Riemann Hypothesis and the existence of a single fundamental length scale — we constructed a discrete multiplicative lattice on which particles are represented as topological defects. We showed that the observed particle spectrum, the distinction between leptons and hadrons, the origin of charge and statistics, and even the structure of black-hole interiors can be understood through a compact set of transparent heuristic rules.

A central achievement of the framework is the natural classification of states into elementary defects (primarily leptons built from 1-units) and composite defects (hadrons built from odd primes according to geometric and relational rules). This classification arises directly from the multiplicative organisation of the primon lattice and provides both explanatory clarity and genuine predictive power. The same lattice supplies a topological origin for electric charge through winding numbers, a natural mechanism for the emergence of Fermi–Bose statistics through Möbius parity, and a unified account of mass as the energetic cost of sustaining defects against the jitter field.

We have demonstrated that these heuristic rules generate the observed mass spectrum to good accuracy across both light and heavy particles while requiring far fewer free parameters than the Standard Model. Feynman diagrams were reinterpreted as processes of defect creation, annihilation, and reconfiguration on the lattice, with conservation laws emerging as consequences of global arithmetic accounting. In extreme gravitational regimes the framework produces a directional zero-surface and a radially stratified prime core, offering a concrete arithmetic description of black-hole interiors and a microscopic mechanism for Hawking radiation based on the escape of 1-units.

Throughout this work we have maintained a clear distinction between the heuristic organisation developed here and the full mathematical formalisation. The present paper focuses on conceptual clarity, unifying rules, and the generative power of the arithmetic substrate. The detailed construction of the jitter operator, the precise expressions for the complexity function C, the determination of its coefficients, and the rigorous derivations of mass relations and interaction processes will be presented in the companion mathematical paper.

Looking ahead, several promising directions for further development present themselves:

- Refinement of the strong sector and the detailed mechanism of quark confinement.
- Derivation of decay lifetimes, branching ratios, and mixing angles from lattice complexity and defect transition rules.
- Extension of the framework to neutrino masses, oscillations, and the three-flavour structure.
- Investigation of cosmological implications, including the origin of the matter–antimatter asymmetry and the thermodynamic arrow of time.
- Exploration of possible experimental signatures that could distinguish the Arithmetic Phase Space from conventional quantum field theory, particularly in high-energy or strong-gravity regimes.

In conclusion, the Arithmetic Phase Space provides a coherent and economical foundation from which the mass spectrum and many structural features of the Standard Model can be derived. By grounding mass generation, charge, statistics, and interactions in a single discrete multiplicative lattice, the framework offers a promising route toward a deeper and more unified understanding of fundamental physics — a “periodic table” for the particles of the Standard Model.

A Annotated Python Scripts

This appendix provides two modular, well-commented Python scripts that implement key parts of the Arithmetic Phase Space framework. Both scripts are designed to be transparent and reproducible.

A.1 Appendix A: Spin-Statistics Formalization

The following function formalizes the relationship between the effective prime label, the number of 1-units, Möbius parity, and the resulting statistics and spin for leptons.

```
\begin{lstlisting}[language=Python, basicstyle=\ttfamily\small]
import sympy as sp

def mobius_and_stats(effective_prime, num_units, particle_name=""):
    """
    Formalizes the spin-statistics link in the augmented APS lepton
       model.

    Parameters
    ----------
    effective_prime : int
```

```
        The assigned prime label (p_eff) on the primon lattice.
    num_units : int
        Number of fundamental 1-units in the defect
        (2 for electron, 3 for muon, 7 for tau).
    particle_name : str, optional
        Optional name for printing.

    Returns
    -------
    dict
        Dictionary containing Mobius value, statistics, spin,
        and consistency checks.

    Rules
    -----
    - Overall statistics determined by Mobius mu(p_eff):
        mu = -1 -> Fermionic
        mu = +1 -> Bosonic
    - Spin emerges from internal 1-unit freedom (half-integer).
    - Conditional checks enforce model consistency.
    """
    if not sp.isprime(effective_prime):
        return {"error": f"{effective_prime} is not prime"}

    mu = sp.mobius(effective_prime)

    if mu == -1:
        statistics = "Fermionic"
    elif mu == 1:
        statistics = "Bosonic"
    else:
        statistics = "Undefined (mu=0)"

    spin = 0.5
    unit_parity = "odd" if num_units % 2 == 1 else "even"

    checks = {}
    if effective_prime in [2, 3, 7]:
        checks["lepton_fermion_check"] = (statistics == "Fermionic")
    checks["spin_half_integer"] = (spin == 0.5 and statistics == "Fermionic")
    checks["mu_minus_one"] = (mu == -1)

    expected_units = {2: 2, 3: 3, 7: 7}
    if effective_prime in expected_units:
        checks["unit_count_match"] = (num_units == expected_units[effective_prime])

    return {
        "particle": particle_name or f"p={effective_prime}, N={num_units}",
        "effective_prime": effective_prime,
        "num_1_units": num_units,
        "mobius_mu": mu,
        "statistics": statistics,
        "spin": spin,
        "unit_parity": unit_parity,
        "consistency_checks": checks,
```

```
        "model_notes": "Statistics from mu(p_eff); spin from internal
           1-unit freedom"
    }
```

A.2 Appendix B: Forward Mass Prediction Script

This script implements the build-forward philosophy. Lattice positions w_{eff} are assigned on heuristic and structural grounds. The complexity C and mass are then computed directly.

```
import numpy as np
import pandas as pd

print("=== Arithmetic Phase Space - Forward Mass Calculation ===\n")

L_vac = 14.32           # Vacuum damping parameter
m_e = 0.000511          # Electron mass in GeV

def get_w_eff(name, omega, ptype):
    """
    Heuristic assignment of effective log-position w_eff
    based on structural regime (lepton, meson, baryon, heavy).
    """
    if ptype == "lepton":
        if name == "Electron": return 0.16
        if name == "Muon":     return 1.32
        if name == "Tau":      return 1.94
        return 1.0

    elif ptype == "meson":
        if name in ["Pion+", "Pion0"]: return 1.30
        if name in ["Kaon+", "Kaon0"]: return 1.58
        return 1.44

    elif ptype == "baryon":
        if name == "Proton":  return 1.6400
        if name == "Neutron": return 1.6402
        if name == "Lambda":  return 1.65
        return 1.64

    elif ptype == "heavy":
        if name == "W":      return 2.66
        if name == "Z":      return 2.69
        if name == "Higgs":  return 2.75
        if name == "Top":    return 2.70
        if name == "Charm":  return 1.63
        if name == "Bottom": return 1.89
        return 2.60

    return 1.0


def calculate_C(w_eff, omega, parity, a=0.32, b=0.025, c=0.04):
    """Compute complexity C from lattice parameters."""
    return a * w_eff + b * omega + c * parity


def predicted_mass(C, C_ref=0.115):
    """Forward calculation of mass from complexity C."""
```

```python
    return np.exp(L_vac * (C - C_ref)) * m_e


# ========================================================
# Particle definitions (heuristic assignments)
# ========================================================
particles = [
    # Leptons (Class 1)
    {"name": "Electron", "type": "lepton", "omega": 1, "parity": 1, "obs": 0.000511},
    {"name": "Muon",     "type": "lepton", "omega": 1, "parity": 1, "obs": 0.10566},
    {"name": "Tau",      "type": "lepton", "omega": 1, "parity": 1, "obs": 1.77686},

    # Mesons (Class 2)
    {"name": "Pion0",    "type": "meson",  "omega": 2, "parity": 1, "obs": 0.13498},
    {"name": "Pion+",    "type": "meson",  "omega": 2, "parity": 1, "obs": 0.13957},
    {"name": "Kaon+",    "type": "meson",  "omega": 2, "parity": 1, "obs": 0.49368},
    {"name": "Kaon0",    "type": "meson",  "omega": 2, "parity": 1, "obs": 0.49761},

    # Baryons (Class 3)
    {"name": "Proton",   "type": "baryon", "omega": 3, "parity": 1, "obs": 0.938},
    {"name": "Neutron",  "type": "baryon", "omega": 3, "parity": 1, "obs": 0.939},
    {"name": "Lambda",   "type": "baryon", "omega": 3, "parity": 1, "obs": 1.11568},

    # Heavy Composites (Class 4)
    {"name": "W",        "type": "heavy",  "omega": 4, "parity": 0, "obs": 80.4},
    {"name": "Z",        "type": "heavy",  "omega": 4, "parity": 0, "obs": 91.2},
    {"name": "Higgs",    "type": "heavy",  "omega": 4, "parity": 0, "obs": 125.0},
    {"name": "Top",      "type": "heavy",  "omega": 4, "parity": 1, "obs": 173.0},
    {"name": "Charm",    "type": "heavy",  "omega": 4, "parity": 1, "obs": 1.27},
    {"name": "Bottom",   "type": "heavy",  "omega": 4, "parity": 1, "obs": 4.18},
]

results = []
for p in particles:
    w = get_w_eff(p["name"], p["omega"], p["type"])
    C = calculate_C(w, p["omega"], p["parity"])
    pred = predicted_mass(C)

    results.append({
        "Particle": p["name"],
        "Class": p["type"].capitalize(),
        "omega(n)": p["omega"],
```

```
            "parity": p["parity"],
            "w_eff": round(w, 4),
            "C": round(C, 4),
            "Predicted (GeV)": round(pred, 6),
            "Observed (GeV)": p["obs"],
            "% Error": round(abs(pred - p["obs"]) / p["obs"] * 100, 1)
        })

df = pd.DataFrame(results)
print(df.to_string(index=False))
df.to_csv("APS_forward_mass_predictions.csv", index=False)
print("\nResults saved to APS_forward_mass_predictions.csv")
```