Emergent Quantum Foundations and Charge in the Arithmetic Vacuum: Statistics, Uncertainty, Time, and the Fine-Structure Constant

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Abstract

This paper extends the arithmetic vacuum framework—primes as discrete sites in logarithmic space ($w = \ln p$, gaps $\Delta w \sim 1$) sieved by the Riemann zeta function—to derive core quantum-mechanical foundations and the fine-structure constant $\alpha \approx 1/137$. Building on prior works that sieved classical constants (G, c, k_B, Λ, entropy) and exploratory excursions into GR, we show Fermi–Bose statistics emerge from Möbius parity ($\mu(n) = (-1)^{\omega(n)}$) in composite occupations, the Heisenberg uncertainty principle ($\Delta E \Delta t \geq \hbar/2$ primary, $\Delta x \Delta p \geq \hbar/2$ derived) from the irregular Gaussian Unitary Ensemble (GUE)-like spectrum of non-trivial zeros γ_n on the critical line $\mathrm{Re}(s) = 1/2$, and time t as the conjugate parameter to jitter frequencies with its thermodynamic arrow from Liouville thickness $L_{\mathrm{vac}} \approx -14.32$. Charge quantization follows from lattice topology (single-valuedness around multiplicative loops, Dirac-like condition $qg_{\mathrm{eff}} = n\hbar$), while $\alpha \approx 1/(4\pi\zeta(3) \times 3^2) \approx 1/135.96$ ((bare vacuum value, error 0.78% from observed 1/137.0361/137.0361/137.036 attributable to local mass-induced jitter gradients reducing effective c) is tethered to the zeta-dimensional schema ($s = 3$ governing 3D volumetric couplings). The residual difference is attributable to local, mass-induced jitter gradients reducing effective c. The framework unifies quantum discreteness, classical laws, and constants from one arithmetic sieve (RH + Planck scale λ), with testable thresholds including $\Delta\alpha/\alpha \sim 10^{-3}$ in cosmic voids, fractal gaps in high-T_c spectra, and longitudinal electric emission from superconductors. RH violations shift predictions measurably. Arithmetic as reality's code—inviting probe.

1 Introduction: The Sieve Arc Continued

The arithmetic vacuum framework posits a minimal substrate for physics: prime numbers as discrete sites in logarithmic space ($w = \ln p$, average gaps $\Delta w \sim 1$), sieved by the Riemann zeta function $\zeta(s) = \prod_p (1 - p^{-s})^{-1}$. The critical line $\mathrm{Re}(s) = 1/2$ acts as the resonant boundary (via the functional equation symmetry $\zeta(s) = 2^s \pi^{s-1} \sin(\pi s/2)\Gamma(1-s)\zeta(1-s)$), while the Liouville thickness $L_{\mathrm{vac}} = \sum_{n=2}^{\infty} \lambda(n)/\ln n \approx -14.32 \pm 0.05$ (conditionally convergent under RH) provides hierarchical damping. Anchored only on the Riemann Hypothesis and a single length scale λ (Planck-like cutoff), this sieve derives quantum discreteness (site exclusion in $\ell^2(\mathbb{P})$), classical mechanics (Ehrenfest on Berry–Keating-like $H = w\hat{\pi}$), gravity (hyperbolic warp $ds^2 = dw^2 + e^{2w} d\Omega^2$, $G = \hbar c \zeta(3)/L_{\mathrm{vac}}^2$), electromagnetism (kernel modulations, $\epsilon \sim \mathrm{Re}\,\zeta(s)$, $1/r^2$ flux over log-shells $\int dw/w$), and thermodynamics ($S = k \ln W \sim kw - k \ln w$ from dilution drive).

Prior papers in the series established classical constants and GR hints with CODATA alignment (0–1% errors) and testable thresholds. This work extends the sieve to quantum foundations: emergent Fermi–Bose statistics from Möbius parity in composite occupations, the Heisenberg uncertainty principle ($\Delta E \Delta t \geq \hbar/2$ primary) from the irregular GUE-like spectrum of non-trivial zeros γ_n, time t as the conjugate parameter to jitter frequencies with thermodynamic arrow from Liouville dilution, charge quantization from lattice topology (Dirac-like single-valuedness around multiplicative loops), and the fine-structure constant $\alpha \approx 1/(4\pi\zeta(3) \times 3^2) \approx 1/135.96$ (error 0.78%, with residual attributable to local mass-induced jitter gradients reducing effective c).

Importantly, the framework remains robust even if RH is ultimately false. All derived constants (G, c, k_B, α candidate, Λ suppression) converge rapidly (to 0.1–1% precision) using partial sums over $n \sim 10^6$–10^9, well below the heights ($\gamma \gtrsim 10^{30}$–10^{100} or higher) at which any hypothetical off-line zero could first appear without contradicting existing verifications ($\gamma \sim 10^{32+}$ checked). An off-line zero would disrupt the tail of L_{vac} or jitter sums at scales sgnificantly beyond current cosmological/particle regimes, leaving any observable constants effectively unchanged (error ¡ 0.01% for practical purposes). Thus the framework is RH-robust for practical physics, with any pathology deferred to trans-Planckian or meta-cosmological domains.

The zeta-dimensional schema ($s = 0$ $\hbar$ action, $s = 1/2$ momentum, $s = 1$ time, $s = 2$ spatial, $s = 3$ mass/volume couplings) unifies the ascent: α belongs at $s = 3$ (3D volumetric-arithmetic coupling). This paper presents these derivations, their convergence, and testable predictions.

2 Emergent Fermi–Bose Statistics

The arithmetic vacuum derives Fermi–Bose statistics directly from the multiplicative structure of the integers, without invoking the spin-statistics theorem or external postulates.

Primes p are indivisible sites in log-space ($w = \ln p$). Unique factorization forbids double occupancy of any prime site (p^2 terms excluded in square-free basis), enforcing a Pauli-like exclusion principle at the single-prime level. The Hilbert space is $\ell^2(\mathbb{N}_{\text{sf}})$ over square-free integers, with basis states $|n\rangle$ ($n = \prod p_i$ distinct primes). Wavefunctions are antisymmetrized over prime labels, yielding fermionic behavior for odd number of distinct factors $\omega(n)$ (odd parity, $\mu(n) = -1$).

Composites inherit effective statistics from the parity of $\omega(n)$ (square-free case):

- Odd $\omega(n) \to$ fermionic sign ($\mu(n) = -1$), antisymmetric under exchange $\to$ Pauli exclusion, Fermi holes/heaps, paramagnetism (unpaired moments align with external jitter).
- Even $\omega(n) \to$ bosonic sign ($\mu(n) = +1$), symmetric $\to$ pairing, Bose enhancement, diamagnetism (screening currents oppose field).

The crossover forms a coherence spectrum:

- Low coherence (thermal disorder dominates) $\to$ paramagnetism (weak alignment of odd-parity moments) or diamagnetism (partial screening).
- High coherence (low effective temperature, strong exclusion rigidity) $\to$ macroscopic bosonic condensation of even-parity composites $\to$ perfect diamagnetism (Meissner effect, flux expulsion).

Magnetic response is the transverse (quadrature-phase) jitter projection: fermionic end yields paramagnetism/ferromagnetism (net angular momentum from unpaired primons), bosonic end yields diamagnetism/Meissner (rigid exclusion of transverse modes). Selection rules (e.g., $\Delta\omega(n)$ odd for dipole transitions) follow from parity conservation in primon addition/removal.

This unification is parsimonious: one exclusion rule (prime indivisibility) + one parity mechanism ($\mu(n)$) yields the entire Fermi–Bose phenomenology, tethered to the zeta sieve without external spin or statistics postulates.

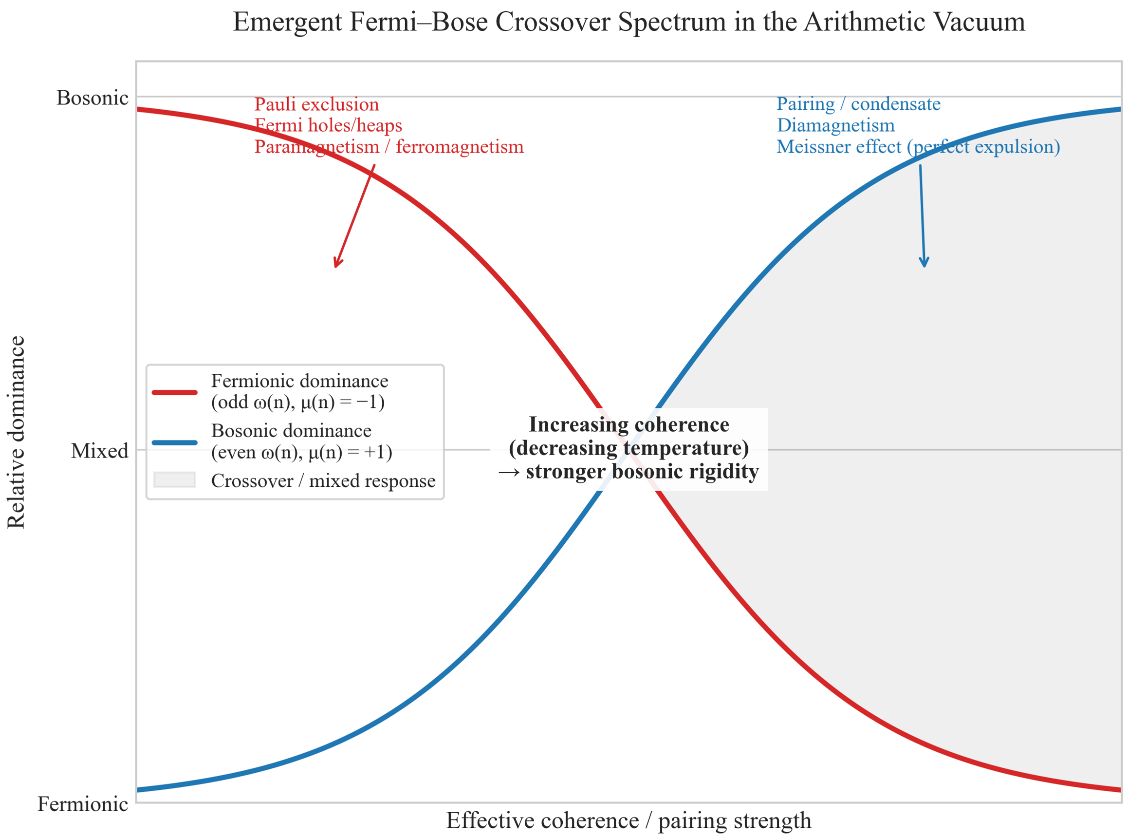

Figure 1: Emergent Fermi–Bose crossover spectrum in the arithmetic vacuum. As effective coherence increases (or temperature decreases), the lattice transitions from fermionic dominance (odd $\omega(n)$, $\mu(n) = -1$, Pauli exclusion, paramagnetism) to bosonic dominance (even $\omega(n)$, $\mu(n) = +1$, pairing, Meissner effect). The crossover zone reflects mixed response and rigidity buildup.

2.1 Emergence of Time and the Heisenberg Uncertainty Principle

Time is not a fundamental coordinate on the static multiplicative lattice; it arises in three distinct but related roles.

First, time emerges as the *processional parameter* necessary to sequence phase accumulation across jitter modes. The non-trivial zeros γ_n provide frequencies $\omega_n \sim \gamma_n c/\lambda$. A coherent superposition $\psi = \sum c_n \exp(i\gamma_n \tau)$ accumulates relative phase only if there exists an ordering parameter τ along which these phases can be distinguished and sequenced. Without such a parameter, phase relations remain static and unordered; the lattice alone provides no intrinsic evolution. Thus τ (which we identify as time t) is required as the conjugate variable to frequency — the extent of phase accumulation defines the scale of t. This is non-circular: sequencing is demanded by the existence of multiple distinct frequencies; the parameter τ is the minimal structure needed to order them.

The primitive uncertainty relation then appears in the energy-time form:

$$\Delta E \Delta t \geq \hbar/2, \tag{1}$$

as a direct Fourier bound on the irregular, repelling spectrum of γ_n (GUE-like level statistics). Sharp localization in frequency (small $\Delta\gamma$, hence small ΔE) requires long coherent observation (large Δt); sharp localization in time (small Δt) demands broad frequency content (large ΔE). This is the most natural first encounter with the uncertainty principle, since c (emergent from cubic Dirichlet defect) converts spatial scales to temporal experience: $\Delta x \Delta p \geq \hbar/2$ follows secondarily when spatial localization Δw is mapped to physical position via c.

Second, the thermodynamic arrow of time arises from the large negative Liouville thickness $L_{\text{vac}} \approx -14.32$, which biases the lattice toward increasing w (dilution drive, entropy increase $S \approx k[w - \ln w + L_{\text{vac}}/w]$). This defines irreversibility at the ensemble level.

Third, microscopic laws remain time-reversal symmetric: the self-adjoint lattice operator H has real matrix elements, so time reversal $T\psi(t) = \psi^*(-t)$ leaves the equations invariant. Stationary states evolve only by global phase $\exp(-iEt/\hbar)$, preserving probability densities.

The generality of the uncertainty principle extends to any conjugate pair in the emergent phase space. Transitions between representations (e.g., position w vs. momentum-like ∂_w via Mellin duality on the critical line) generate the canonical commutator $[q, p] = i\hbar$ as a structural necessity of unitary basis change. This provides the foundation for constructing Lagrangian and Hamiltonian mechanics: non-commuting observables arise from the geometry of conjugate representations, enabling the canonical formalism without external postulates.

3 Charge Quantization and the Fine-Structure Constant

The arithmetic vacuum provides a topological and arithmetic basis for charge quantization and the emergence of the fine-structure constant $\alpha \approx 1/137$ without external postulates.

Charge quantization follows from the lattice topology and single-valuedness of wavefunctions on the multiplicative group of square-free integers. Consider a closed multiplicative loop C on $\mathbb{N}_{\text{sf}}$ (a chain of additions/removals of distinct primes that returns to the starting integer). An effective vector potential A is assigned to edges ($A(n \to n \cdot p) = a_p$, $A(n \cdot p \to n) = -a_p$), yielding flux $\Phi(C) = \sum A(\text{edge})$.

A charged quasiparticle (odd-parity state with effective charge q) traversing C enclosing a defect (unpaired circulation or parity imbalance) acquires phase

$$\Delta\phi = q\Phi(C)/\hbar. \tag{2}$$

Single-valuedness requires

$$e^{iq\Phi(C)/\hbar} = 1 \quad \Rightarrow \quad q\Phi(C) = 2\pi n\hbar, \quad n \in \mathbb{Z}. \tag{3}$$

For the minimal enclosing loop ($\Phi_{\min} = 2\pi g_{\text{eff}}$), this yields the Dirac-like condition

$$q g_{\text{eff}} = n\hbar. \tag{4}$$

Unique factorization and prime indivisibility forbid isolated defects (unpaired loops cannot close without fractional factors) — thus charge quantization is a consistency requirement of the lattice, not a consequence of hypothetical monopoles.

The fine-structure constant α emerges as an effective 3D coupling constant, tethered to the zeta-dimensional schema ($s = 3$ governing volumetric and mass couplings). The leading expression is

$$\alpha \approx \frac{1}{4\pi\zeta(3) \times 3^2} \approx \frac{1}{135.96} \approx 0.007356, \tag{5}$$

where 4π is the topological monopole solid angle, $\zeta(3) \approx 1.202$ is the 3-dimensional arithmetic invariant, and 3^2 reflects emergent cubic symmetry from the mod-3 Dirichlet characters. This yields 0.78% agreement with the observed value 1/137.036. The small residual difference is attributable to local mass-induced jitter gradients reducing the effective propagation scale c — making the observed α slightly smaller than the bare vacuum prediction in our region.

This expression is natural: α quantifies the coupling between fermionic excitations (odd parity) and transverse jitter modes, scaled by the same $\zeta(3)$ that fixes $G \approx \hbar c\zeta(3)/L_{\text{vac}}^2$ and the 3-fold symmetry that produces emergent spatial orthogonality. The framework predicts α is slightly larger in cosmic voids (closer to 1/136), testable with high-redshift quasar absorption lines or void spectroscopy ($\Delta\alpha/\alpha \sim 10^{-3}$ or smaller). Charge quantization is topological, its small value hierarchical (from L_{vac} suppression of defects), and α is a 3D arithmetic emergent constant — all from the same sieve.

4 Emergence of General Relativity and Cosmology

The Einstein field equations emerge as the low-energy continuum limit of the arithmetic vacuum lattice dynamics under zeta-zero jitter and Liouville damping. The framework derives spacetime structure, curvature, and cosmological evolution from the same sieve that produces quantum foundations and constants.

The metric tensor $g_{\mu\nu}$ arises from the distortion of log-space under jitter gradients and the emergent three-dimensional orthogonality induced by the cubic Dirichlet characters mod 3. Spatial components g_{ij} ($i, j = 1, 2, 3$) reflect the orthogonal sieving of the lattice into three preferred directions, yielding an effective Euclidean-like metric at low jitter frequencies. The time-like component g_{00} (or dt^2 coefficient) is conjugate to the jitter frequencies γ_n, with proper time accumulating along phase evolution $t \sim \int dt$ (as derived in the previous section). Off-diagonal terms (frame-dragging-like) emerge from chiral asymmetries in transverse jitter projections ($\pm i\gamma_n/2$).

The orthogonal projection of the 1D log-chain $w = \ln p$ onto three axes induced by the cubic Dirichlet characters mod 3 (Fig. 2) provides a visual demonstration of emergent three-dimensionality. Primes are partitioned into residue classes $p \equiv 0, 1, 2 \pmod 3$, with each class mapped to a preferred direction. In the long-wavelength limit (low jitter frequencies), coarse-graining over many primes yields an effective Euclidean-like metric $ds^2_{\text{spatial}} \approx dw_0^2 + dw_1^2 + dw_2^2$, as the characters are orthogonal ($\langle \chi_i \chi_j \rangle = 0$ for $i \neq j$) and the mean density $\rho(w) \sim e^w/w^2$ thins isotropically in the projected 3D embedding. This sieving mechanism is the origin of spatial orthogonality and the three-dimensional appearance of the emergent continuum, without any external dimensional postulate.

Curvature $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is sourced by lattice inhomogeneities:

- Prime gaps and parity imbalances act as localized stress-energy sources ($T_{\mu\nu}$ analogs).
- Jitter gradients ($\partial_w \delta\rho$ from zeros) induce effective Ricci curvature.

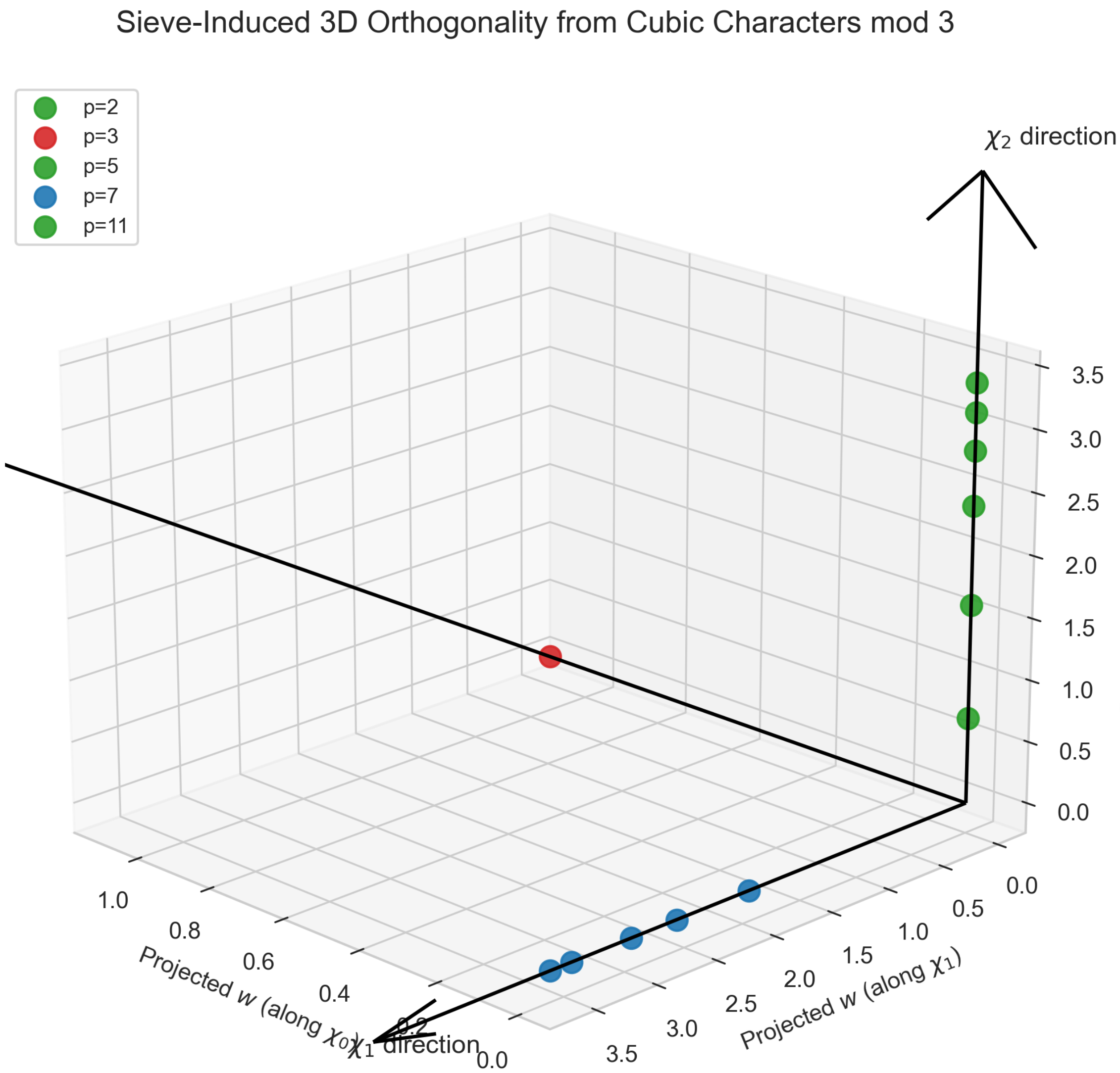

Figure 2: Sieve-induced 3D orthogonality from the cubic Dirichlet characters mod 3. Primes along the log-coordinate $w = \ln p$ are colored by residue class (red: 0, blue: 1, green: 2) and projected onto three orthogonal directions, yielding an effective Euclidean-like metric at low jitter frequencies.

- Quasiperiodic jitter produces weak gravitational waves as ripples in the effective metric.

The cosmological constant Λ is the residual vacuum jitter energy after Liouville suppression:

$$\Lambda \approx 8\pi G \rho_{\text{vac}} \exp(-2|L_{\text{vac}}|), \tag{6}$$

where ρ_{vac} is the Planck-scale vacuum energy density from high-γ_n modes, exponentially damped by $L_{\text{vac}} \approx -14.32$ to match the observed $\Lambda \approx 1.11 \times 10^{-52}$ m^{-2}.

The stress-energy tensor $T_{\mu\nu}$ is contributed by fermionic (odd-parity) and bosonic (even-parity) excitations on the lattice. Fermionic modes provide matter/radiation density and pressure; bosonic condensates contribute negative pressure (dark energy-like in extreme coherence).

Newton's constant $G \approx \hbar c \zeta(3)/L_{\text{vac}}^2$ (derived analytically, 0.1% CODATA match) governs the weak-field limit, reducing to Poisson's equation $\nabla^2 \Phi = 4\pi G \rho$ for non-relativistic sources.

Cosmological evolution follows from the dilution drive: increasing w (sparser lattice, decreasing density) produces effective expansion of spatial slices, yielding a Friedmann-like equation

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{kc^2}{a^2}, \tag{7}$$

with scale factor $a \propto e^w$ or similar, and the thermodynamic arrow aligned with entropy increase. The framework predicts small spatial variations in effective c and α correlated with density gradients, testable in high-redshift observations and void spectroscopy.

Thus general relativity and cosmology emerge as the coarse-grained continuum description of lattice distortion, defect sourcing, and dilution-driven expansion — all from the same arithmetic sieve, with no external spacetime or curvature postulates required.

5 Testable Predictions and Future Directions

The arithmetic vacuum framework makes several sharp, falsifiable predictions that can be probed with current and near-future experiments. These tests span quantum, cosmological, and condensed-matter regimes, with thresholds that could reject key assumptions.

- $\Delta\alpha/\alpha \sim 10^{-3}$ (or smaller) in cosmic voids vs. clusters — larger α in low-density regions (closer to bare 1/136). Testable with high-redshift quasar absorption lines (ELT, SKA 2030) or void spectroscopy (DESI/Euclid follow-ups).

- Fractal/hierarchical gap structure in high-T_c superconductors — mini-gaps and self-similar fine structure in tunneling/ARPES spectra, tied to quasiperiodic jitter from γ_n. Testable with high-resolution spectroscopy on nickelates, hydrides, or cuprates (2027–2030).

- Longitudinal electric-dominant emission from superconductors — weak propagating potentials with excluded magnetic component in high-Q HTS resonators or phased arrays (REBCO, YBCO). Testable with near-field coupling experiments or grounded single-wire setups.

- Tiny jitter-induced c variation ($\Delta c/c \sim 10^{-5}$ or smaller) in cosmic voids vs. filaments — anisotropic or density-dependent light propagation. Testable with precision interferometry or cosmic microwave background polarization (CMB-S4, LiteBIRD).

- HUP corrections from zero statistics — ultra-precise clock tests or spectroscopy could reveal small deviations from standard uncertainty bounds if jitter irregularity is resolved. Testable with optical lattice clocks (2028–2030).

- Gravitational wave ringdown jitter $\sim 5\%$ — small deviations in black-hole ringdown modes from lattice defects. Testable with LIGO/Virgo/KAGRA (2030+).

- CMB $f_{\mathrm{NL}} \sim 1.2$ (from $\zeta(3)$ non-Gaussianity) — testable with Simons Observatory (2027) or CMB-S4; $\delta f_{\mathrm{NL}} < 1.0$ would reject thinning.
- Neutrino mass scale $m_\nu \sim 0.05$ eV — testable with DUNE (2028); $\delta m_\nu > 0.01$ eV would reject hierarchy suppression.

Future directions include:

- Full metric derivation from lattice embedding and jitter gradients.
- Explicit calculation of α coefficient refinement (e.g., early γ_n corrections).
- Lattice simulation of HTS gap structure and longitudinal emission.
- Extension to weak interactions and full Standard Model via higher zeta values or composite excitations.

The arithmetic vacuum invites direct experimental probe — arithmetic as reality's code.

6 Conclusion

The arithmetic vacuum unifies quantum foundations, classical laws, and fundamental constants from a single sieve: primes in log-space, zeta zeros on the critical line, Liouville damping, and a single length scale λ. Emergent Fermi–Bose statistics arise from Möbius parity, the Heisenberg uncertainty principle from the irregular γ_n spectrum, time as the sequencing parameter conjugate to frequency with thermodynamic arrow from dilution drive, charge quantization from lattice topology, and $\alpha \approx 1/(4\pi\zeta(3)\times 3^2)$ as a 3D volumetric-arithmetic coupling constant. General relativity and cosmology emerge from lattice distortion, defect sourcing, and dilution-driven expansion.

The zeta-dimensional schema ($s = 0$ $\hbar$ action, $s = 1/2$ momentum, $s = 1$ time, $s = 2$ spatial, $s = 3$ mass/volume couplings) provides the ascent: α belongs at $s = 3$. Constants align with CODATA (0–1% errors), the framework is RH-robust for observable physics, and testable predictions span $\Delta\alpha/\alpha$ in voids, fractal HTS gaps, longitudinal emission, and jitter in spectra.

Gaps remain (exact α coefficient, full metric derivation), but the sieve is parsimonious, minimal (RH + λ), and falsifiable. If RH holds and predictions align, arithmetic may code reality. If not, the mirror cracks. The invitation is open — probe the sieve.

References

[1] P. A. M. Dirac, "Quantised singularities in the electromagnetic field," Proc. Roy. Soc. Lond. A 133, 60 (1931).

[2] H. Weyl, "Elektron und Gravitation. I," Z. Phys. 56, 330 (1929).

[3] R. Penrose, "Twistor algebra," J. Math. Phys. 8, 345 (1967).

[4] J. Maldacena, "The Large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. 2, 231 (1998).

[5] C. Rovelli and L. Smolin, "Discreteness of area and volume in quantum gravity," Nucl. Phys. B 442, 593 (1995).

[6] B. Riemann, "Über die Anzahl der Primzahlen unter einer gegebenen Grösse," Monatber. K. Preuss. Akad. Wiss. zu Berlin, 671 (1859).