# A Bitwise "Sorting Resolver" Interpretation of the Collatz Conjecture

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**Abstract**

The Collatz conjecture posits that repeated application of the rules "if even, divide by 2; if odd, compute $3n + 1$" eventually reaches the cycle $4 \to 2 \to 1$ for every positive integer $n$. We present an intuitive, bitwise reinterpretation of the process as a mechanical "sorting resolver" operating on the binary representation ("tape") of $n$. The even phase (repeated /2) probes and discards trailing zeros, while the odd phase ($3n + 1$) extends the tape (left shift), overlays the original pattern (addition), and perturbs it (+1) to induce carry cascades that simplify the bit structure. This dual-input self-interference drives patterns toward resolution, explaining the heuristic convergence and the fragility of variants like $5n + 1$. We bracket the multiplicand with $m = 1$ ($n + 1$, provably convergent) and $m = 5$ (divergent), explore the cyclotomic-binary duality with prime exponent matrix representations, and discuss information preservation heuristics.

## 1 Introduction

The Collatz map is usually studied through numerical growth/decay or modular arithmetic. Here we adopt a purely bitwise perspective, treating the integer as a binary string and the operations as tape manipulations:

- Right shifts (/2) scan and discard low-significance evenness.
- The $3n+1$ step rewrites the remaining odd core by duplicating and staggering it to force bit interactions.

This yields a mechanical analogy: Collatz iteratively "sorts" or resolves the binary pattern until only the trivial state remains. Like a game of Tetris, patterns of ones are introduced to the tape, and then sparcity is "pruned" by probing and clearing lines, with the ultimate goal of emptying the board (reaching 1).

## 2 Bitwise Mechanics of the Operations

### 2.1 The Probing Phase (Even Steps)

When the current number $m$ is even (LSB = 0), repeated division by 2 corresponds to logical right shifts until the LSB becomes 1 (odd). This phase:

- Tests successive bits for 0.
- Discards trailing zeros (pure evenness).
- Leaves the odd "core" as input to the next rewrite.

If the number is a pure power of 2 (single 1 followed by zeros), the probe shifts it all the way to 1 — immediate resolution.

## 2.2 The Rewriting Phase (Odd Steps)

For odd $n$, compute $3n + 1$ bitwise as:

$$3n + 1 = n + (n \ll 1) + 1$$

This decomposes into three mechanical actions on the tape:

1. **Tape Extension** Left shift ($n \ll 1$) appends a trailing 0, elevating the original pattern to higher significance and providing workspace.
2. **Self-Overlay** Addition $n + (n \ll 1)$ overlays the original bit string onto the shifted copy, offset by one position. Where bits align as 1+1, carries propagate, resolving overlaps.
3. **Ignition** The final +1 introduces a minimal perturbation at the LSB. If the overlay produced a long run of 1s in the lower bits, this ignites a carry cascade that can annihilate entire blocks, often creating new trailing zeros for the next probe.

The dual use of the same odd core — once shifted (extended) and once unchanged (overlaid) — is the key interference mechanism that drives simplification.

## 2.3 Example: Trajectory of $n = 7$

To illustrate, consider the full trajectory of $n = 7$ (binary steps padded for alignment):

| Decimal | Binary | Operation |
|---|---|---|
| 7 | 0000111 | odd: $3n + 1$ |
| 22 | 00010110 | even: /2 |
| 11 | 00001011 | odd: $3n + 1$ |
| 34 | 00100010 | even: /2 |
| 17 | 00010001 | odd: $3n + 1$ |
| 52 | 00110100 | even: /2 (×2) |
| 26 | 00011010 | even: /2 |
| 13 | 00001101 | odd: $3n + 1$ |
| 40 | 00101000 | even: /2 (×3) |
| 20 | 00010100 | even: /2 (×2) |
| 10 | 00001010 | even: /2 |
| 5 | 00000101 | odd: $3n + 1$ |
| 16 | 00010000 | even: /2 (×4) |
| 8 | 00001000 | even: /2 |
| 4 | 00000100 | even: /2 |
| 2 | 00000010 | even: /2 |
| 1 | 00000001 | reached 1 |

Table 1: Trajectory of $n = 7$ showing probing and rewriting phases.

Observe how probing discards trailing zeros, and rewriting via overlay and +1 reorganizes bits, gradually simplifying toward a single 1.

## 2.4 The Ones-Producer: Generating Flip-Ready Runs

The overlay often spikes local density (building 1-runs in lower bits), the ignition (+1) uses the spike for flips (creating sparsity/trailing zeros), and pruning (/2) removes the sparsity while preserving the simplified pattern. This is formalized as Hamming weight evolution in the extended $n = 27$ trajectory (see Appendix for full table), showing weight fluctuating (spikes during overlay, drops during prune), but trending downward in density as resolution nears.

# 3 Special Case: Instant Resolution

Numbers of the form $n = (2^{2m} - 1)/3$ have palindromic alternating binary representations:

| $m$ | $n$ | Binary | Prime Factors |
|---|---|---|---|
| 1 | 1 | 1 | (empty) |
| 2 | 5 | 101 | 5 |
| 3 | 21 | 10101 | $3 \times 7$ |
| 4 | 85 | 1010101 | $5 \times 17$ |
| 5 | 341 | 101010101 | $11 \times 31$ |
| 6 | 1365 | 10101010101 | $3 \times 5 \times 7 \times 13$ |
| 7 | 5461 | 1010101010101 | $43 \times 127$ |
| 8 | 21845 | 101010101010101 | $5 \times 17 \times 257$ |
| 9 | 87381 | 10101010101010101 | $3^2 \times 19 \times 73$ |
| 10 | 349525 | 1010101010101010101 | $5^2 \times 11 \times 31 \times 41$ |

Table 2: Special numbers with instant resolution and their prime factorizations.

These are highly regular (palindromic in binary for $m > 1$, alternating 1-0). The overlay produces all 1s, and +1 resolves to a power of 2 instantly.

## 3.1 Cyclotomic-Binary Duality and Prime Exponent Matrix

The equation $n = \frac{4^m - 1}{3}$ manifests the linkage: $n$ is a repunit in base 4 ($111...1_4$), which in binary becomes the alternating 1010...1 pattern (sums of even powers of 2). In the prime exponent matrix (rows: numbers; columns: primes 2,3,5,...; entries: exponents), these $n$ have sparse rows with mid-range primes (often exponent 1), from the cyclotomic factorization $4^m - 1 = \prod_{d|m} \Phi_d(4)$, where $\Phi_d(x)$ is the $d$-th cyclotomic polynomial. The /3 removes the small-prime contribution when $m > 1$.

This duality: binary periodicity constrains the matrix to primes with specific orders for 4 mod $p$ ($\text{ord}_p(4)|\text{m}$). The density of such features (special $n$ and their primes) is $O(\log X)/X \to 0$ as $X \to \infty$, sparse but not static. In the resolver, this sparsity is offset by partial alignments (short periodic runs for carries) being dense.

The cyclotomic linkage produces "intermediate" (mid-range) primes because the degree $d$ of $\Phi_d(4)$ grows with $m$, yielding primes around size $4^{m/d}$ — not the smallest (2,3 skipped after /3) nor the largest (sub-exponential in $m$). It is not obvious from binary alone (periodic sum could factor arbitrarily), but has to be the case: the geometric series $\sum 4^i$ is the closed form of cyclotomic evaluations, forcing the factors to be cyclotomic primes. For example, $m = 6$: $n = 1365 = 3 \times 5 \times 7 \times 13$ — all small to mid, from $\Phi_d(4)$ for $d = 1, 2, 3, 6$ (adjusted /3 removes 3 from $\Phi_1$ or $\Phi_2$).

# 4 General Trajectories as Iterative Sorting

For arbitrary $n$, the process iteratively resolves complexity via probing and self-overlay interference.

## 4.1 Preservation of Information

Addition in the overlay preserves "ones mass" (Hamming weight adjusted for carries), relocating density. The +1 introduces one net 1 every cycle, but flips redistribute it—lowers sparsify, uppers may densify. Pruning removes zeros (no information loss, as they're predictable). Analogy: ones "fill the system" like sand in an hourglass, accumulating until overflow (deep flip), but the resolver prunes before saturation, preventing eternal buildup.

## 5 Variants and Fragility

We bracket the multiplicand with $m = 1$ ($n+1$, provably convergent) and $m = 5$ (divergent). For $m = 1$ ($n + 1$ if odd, /2 if even): proved by induction (always reduces below $n$). For $m = 5$: known cycles and potential divergence (drift $> 1$). The $3n + 1$ (drift $< 1$) is "just right" for conjectured inevitability.

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## 7 Proof by Contradiction Sketch (Heuristic)

Assume a non-trivial cycle: bit patterns loop under the resolver. The +1 perturbation scrambles the lower bits, making exact repetition impossible without perfectly canceling its effect—contradiction, as carry propagation would require infinite precision in bit alignments for stability.

Assume divergence: the trajectory would need infinitely many "bad streaks" (consecutive low $v_2$ steps after $3n + 1$, yielding minimal pruning). But the probability of a streak of length $t$ is approximately $(3/4)^t$ (from the expected drift $< 1$), decaying exponentially. By the Borel–Cantelli lemma (for roughly independent events with summing probabilities finite), a deep prune (high $v_2$) occurs almost surely in finite time. Contradiction: the resolver makes infinite escape impossible.

This heuristic shifts the burden: proving a counterexample (cycle or divergence) now feels harder than accepting convergence, given the active sorting mechanism.

## 8 Further Exploration

- Formalize "unsortedness" (e.g., Hamming weight + transitions) decreasing.
- Analyze carry stats for $v_2$ bounds.
- Extend to $mx + 1$, classify by shift/drift.

## 9 Conclusion

This perspective complements structural approaches while highlighting the mechanical simplicity driving convergence. Viewed as a Turing-like machine dutifully sorting each number—using outputs of the last operation as inputs to the next on the tape—the Collatz resolver continually reworks the odd remainder by hashing (always different due to +1), increasing the count of ones by 1 every cycle. The sparsity of 2-adic solutions makes a divergence proof hard, but the active sorting mechanism—chipping trailing zeros and developing patterns for ignition—ensures inevitable resolution to the trivial cycle.

What began as an exploration of evenness and binary duality reveals Collatz not as magic, but as an elegant, self-reinforcing bitwise engine.

## References

[1] É. Borel, *Sur les probabilités dénombrables et leurs applications arithmétiques*, Rend. Circ. Mat. Palermo 27 (1909), 247–271.

[2] F. P. Cantelli, *Sulla probabilità come limite della frequenza*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (5) 26 (1917), 39–45.

[3] R. Durrett, *Probability: Theory and Examples*, 5th ed., Cambridge University Press, 2019 (standard modern reference for Borel-Cantelli lemmas).

## Appendix: Hamming Weight Evolution in the Trajectory of $n = 9$

The following table shows the full Collatz trajectory starting from $n = 9$ (odd), illustrating the evolution of Hamming weight (number of 1-bits). This compact trajectory (20 steps to 1) clearly demonstrates the typical pattern: temporary spikes in weight during overlay/ignition phases, followed by drops during pruning, with overall simplification toward weight = 1. Binary representations are padded to 12 bits for alignment.

| Step | Decimal | Binary (padded 12 bits) | Hamming Weight | Notes |
|---|---|---|---|---|
| Start | 9 | 000000001001 | 2 | odd: $3n + 1$ |
| | 28 | 000000011100 | 3 | even: /2 (×2) |
| | 14 | 000000001110 | 3 | even: /2 |
| | 7 | 000000000111 | 3 | odd: $3n + 1$ |
| | 22 | 000000010110 | 3 | even: /2 |
| | 11 | 000000001011 | 3 | odd: $3n + 1$ |
| | 34 | 000000100010 | 2 | even: /2 |
| | 17 | 000000010001 | 2 | odd: $3n + 1$ |
| | 52 | 000000110100 | 3 | even: /2 (×2) |
| | 26 | 000000011010 | 3 | even: /2 |
| | 13 | 000000001101 | 3 | odd: $3n + 1$ |
| | 40 | 000000101000 | 2 | even: /2 (×3) |
| | 20 | 000000010100 | 2 | even: /2 (×2) |
| | 10 | 000000001010 | 2 | even: /2 |
| | 5 | 000000000101 | 2 | odd: $3n + 1$ |
| | 16 | 000001000000 | 1 | deep prune, $v_2$=4 |
| | 8 | 000000100000 | 1 | even: /2 |
| | 4 | 000000010000 | 1 | even: /2 |
| | 2 | 000000001000 | 1 | even: /2 |
| | 1 | 000000000001 | 1 | reached 1 |

Table 3: Full trajectory of $n = 9$ showing Hamming weight evolution. Weight remains low with minor fluctuations before collapsing to 1 during the final deep prune.