The EPR Paradox and Bell Inequalities: A Foundation for Quantum Completeness

Ralph Hassall

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Abstract

This paper resolves the EPR paradox and reframes Bell inequalities. It does so via a single entangled wavefunction with coordinated boundary conditions, descriptively selecting one observable per measurement. This resolves EPR and reframes Bell inequalities as confirmation of quantum completeness, without requiring hidden variables.

1 What EPR Says: The Question of Completeness

In their 1935 paper, Einstein, Podolsky, and Rosen (EPR) argued that quantum mechanics is incomplete because it does not assign definite values to all observables, failing to describe all elements of physical reality. They define an element of physical reality as a quantity predictable with probability unity without disturbing the system.

EPR highlight this issue with non-commuting operators, such as position (x) and momentum (p), satisfying the canonical commutation relation $[x, p] = i\hbar$. For an entangled two-particle system (e.g., correlated positions and momenta), measuring one particle's observable (e.g., position) reduces the wave function to an eigenstate, yielding a definite value for the other particle's corresponding observable (e.g., P(x_1=x_2)=1). The central issue confronted by EPR is that there are apparently two different wavefunctions that describe a remote particle which has not been influenced in any physical manner.

This idea was subsequently expanded, e.g. Bohm and Aharonov, to correlated spin systems. For the singlet state $\psi = (|\uparrow_z\downarrow_z\rangle - |\downarrow_z\uparrow_z\rangle)/\sqrt{2}$, measuring both spins along the same axis (e.g., z) yields perfect anticorrelation (e.g., if particle I is $\uparrow_z$, particle II is $\downarrow_z$, with probability 1). This satisfies EPR's reality criterion at cardinal points ($\theta = 0°$), resembling eigenstates post-measurement, but quantum mechanics does not assign definite values for all directions, prompting an incompleteness claim.

2 What Bell Says About EPR: The Hidden Variable Test

2.1 The Bell Framework and Its Assumptions

John Bell's 1964 theorem tests an idea to explain EPR incompleteness: the hidden variable hypothesis. For the singlet state, quantum mechanics predicts the spin correlation $E(\mathbf{a}, \mathbf{b}) = \langle\psi|(\boldsymbol{\sigma}\cdot\mathbf{a})\otimes(\boldsymbol{\sigma}\cdot\mathbf{b})|\psi\rangle = -\mathbf{a}\cdot\mathbf{b} = -\cos\theta$, where θ is the angle between measurement directions $\mathbf{a}$ and $\mathbf{b}$. In the CHSH inequality, quantum mechanics predicts $|S| = 2\sqrt{2} \approx 2.828$ at non-cardinal angles (e.g., 22.5°, 45°, 67.5°), violating the classical bound $|S| \leq 2$.

Bell assumes a local hidden variable model in which outcomes are predetermined by a variable λ with probability density $\rho(\lambda)$: $E(\mathbf{a}, \mathbf{b}) = \int A(\mathbf{a}, \lambda)B(\mathbf{b}, \lambda)\rho(\lambda)d\lambda$, with $A, B = \pm 1$. Locality ensures A is independent of $\mathbf{b}$, and vice versa, yielding the Bell inequality $|S| \leq 2$. Experiments confirm $|S| = 2\sqrt{2}$, supporting quantum mechanics and ruling out local hidden variables. To undertake this proof, Bell makes a constraint on the axis of rotation θ, which removes the hidden variable probability density term. Violations of the inequality are found with variation in angular dependence.

Bell does not use observables with the classical non-commuting operator relationship as specified in the EPR paper and uses one that obeys a cyclic permutation rule. Assessing the singlet spin state also requires the measurement of the same spin state observable, which means that Bell experiments are measuring different values of the same observable (e.g., up). Bell measurements are also undertaken on two separate particles (rather than on one alone in EPR) and this is permitted by commutation rules: i.e., they will always commute, and this means the Bell thought experiment cannot address EPR's central claim. The EPR claim relies on the possibility of two, equally valid, wave functions for the remote particle brought about by action beyond the range of correlation, and this issue is not addressed.

While the theorem tests locality it thus does not directly resolve the incompleteness claim for non-commuting observables. The generalization expressed at the end of the paper—that all observables will obey the Bell result—requires unphysical abstraction to demonstrate.

2.2 Generality of Bell Inequalities: A Range Issue

The correlation $E = -\cos\theta$ is not limited to spin-1/2 systems but arises in any quantum system with rotational symmetry and dichotomic observables, such as photon polarization ($E = -\cos 2\theta$) or momentum correlations in an entangled HD molecule. These yield analogous Bell-like inequalities for these systems, derivable from quantum mechanics' axioms or classical models with a local parameter, but have nothing explicit to do with testing non-locality.

For example, the CHSH form $S = E(\mathbf{a}, \mathbf{b}) + E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) - E(\mathbf{a}', \mathbf{b}') \leq 2$ bounds local models, but QM's (pre-extant) $S = 2\sqrt{2}$ at particular angles already flags this range as problematic for reasons unrelated to non-locality.

This generality implies a range issue: The results would have arisen anyway for any quantum system formulated in a similar manner. I.e., the breach of inequality is a known range issue.

2.3 Bell's Key Innovation: Expressing Hidden Variables

Hidden variables were proposed in response to the EPR paper as a local, deterministic model. Bell's key innovation is devising a mathematical expression for hidden variables in several different systems, and each of these systems is able to explain the observed, experimental results: $E(\mathbf{a}, \mathbf{b}) = \int A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) \rho(\lambda) d\lambda$. With the caveats of preceding sections aside, Bell's method formalizes a system of presenting hidden variables within wave-functions. Bell's formulation enables the derivation of testable inequalities that probe locality without directly addressing non-commuting observables.

3 Why It Does Not Matter: Resolving the EPR Paradox

3.1 The Resolution: No Paradox from Coordinated Boundaries

The EPR paradox resolves because a single entangled wavefunction with coordinated properties (e.g., $q_1 - q_2 = q_0$, $p_1 + p_2 = 0$) ensures only one observable is defined per measurement. Operators on different particles commute (e.g., $[q_1, p_2] = 0$), allowing predictions across subsystems.

Measuring $q_1 = a$ reduces the wave packet to $\psi_2(q_2) = \delta(q_2 - (a - q_0))$, predicting $q_2 = a - q_0$ with certainty. Measuring $p_1 = p$ reduces it to $\phi_2(p_2) = \delta(p_2 + p)$, predicting $p_2 = -p$. Irrelevant amplitudes are removed descriptively as boundary conditions decouple subsubsytems, with no real change (action) in particle II—no physical disturbance.

The measurement on particle I has no influence on particle II's non-commuting observable, allowing its eigenvalue to be retrieved with certainty in a separate experiment. This confirms only one observable is defined per measurement, respecting the uncertainty principle ($\Delta q_2 \cdot \Delta p_2 \geq \hbar/2$). There is also no possibility of different wavefunctions describing the activities of a single particle: the completeness violation of EPR.

3.2 No Paradox, No Non-Locality via Hidden Variables

The wavefunction update is descriptive, removing irrelevant subsystems without physical influence or non-locality. For the singlet, same-axis measurements ($\theta = 0°$) yield definite anticorrelation, satisfying EPR at cardinal points, while non-cardinals remain probabilistic: i.e., these results are not eigenstates, and cannot be disaggregated from orthogonal observables.

To measure singlet states, Bell experiments must use commuting same-direction operators, measuring different eigenvalues of the same observable. This aligns with EPR's reality at cardinal points but not beyond. Violations ($|S| = 2\sqrt{2}$) confirm quantum mechanics' completeness without hidden variables—its predictions and experimental validations stand unaltered. It is the inequality that proves incomplete

4 Conclusion

EPR highlight the incompleteness of quantum mechanics from non-commuting observables and competing wave functions, but coordinated boundaries in the entangled wavefunction resolve the paradox: Descriptive pruning of sub-components in the normal manner selects one observable per context, preserving uncertainty and avoiding non-locality. Bell's inequalities test locality but do not resolve EPR's broader concern and a range issue is identified as a generality in quantum mechanical systems.

EPR conclude: "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description is possible. We believe, however, that such a theory is possible." Based on the EPR argument, this framework presented here affirms the completeness of quantum mechanics, with correlations pre-established in an entangled whole. Further exploration may link to broader interpretations, but the paradox as it stands in EPR dissolves: No hidden variables required.

References

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